7. א. (1)

$f(x)=\frac{2\sin x}{\cos^3 x}\quad,\quad \cos x\neq 0\quad\Rightarrow\quad \boldsymbol{x\neq\frac{\pi}{2}+k\pi}$

(2)

$y=0\quad\Rightarrow\quad \sin x=0\quad\Rightarrow\quad x=\pi k\quad\Rightarrow\quad \boldsymbol{(\pi k,0)}$

נקודות אלו כוללות גם את נקודת החיתוך עם ציר y , $(0,0)$ (כש־ $k=0$).

(3)

$\lim\limits_{x\to(\frac{\pi}{2}+k\pi)}\frac{2\sin x}{\cos^3 x}=\frac{\to\pm 2\cdot 1}{\to 0}=\infty\quad\Rightarrow\quad \boldsymbol{x=\frac{\pi}{2}+k\pi}$

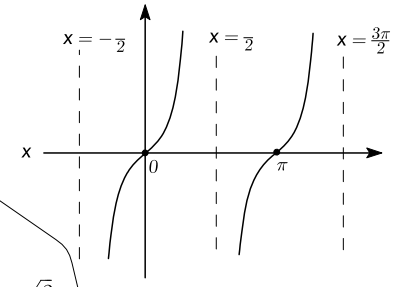
(4)

$f'(x)=\frac{2\cos x\cos^3 x-3\cos^2 x\,(-\sin x)\cdot 2\sin x}{(\cos^3 x)^2}=\frac{2\cos^2 x+6\sin^2 x}{\cos^4 x}$

$\Rightarrow\quad f'(x)>0\quad\forall\{x\neq\frac{\pi}{2}+k\pi\}$

$\Rightarrow\quad \nearrow:\ \boldsymbol{\frac{\pi}{2}+k\pi<x<\frac{\pi}{2}+(k+1)\pi}\ ,\ \searrow:\ \boldsymbol{\emptyset}$

ב.



ג.

$S=\int_0^a\frac{2\sin x}{\cos^3 x}\,dx=\frac{1}{\cos^2 x}\Big|_0^a=\frac{1}{\cos^2 a}-1=1\quad/+1$

$\frac{1}{\cos^2 a}=2\quad\Rightarrow\quad \cos^2 a=\frac{1}{2}\quad\Rightarrow\quad \cos a=\pm\frac{1}{\sqrt{2}}=\pm\frac{\sqrt{2}}{2}\ ,\ 0<a<\frac{\pi}{2}\quad\Rightarrow\quad \boldsymbol{a=\frac{\pi}{4}}$

$\int f[f(u(x))]\cdot u'(x)\,dx=F[u(x)]+c\ ,\ u(x)=\cos x\ ,\ -2\int\frac{1}{\cos^3 x}(-\sin x)\,dx=-2\int\frac{u'(x)}{u^3(x)}dx=-2\cdot\frac{1}{-2u^2}+c=\frac{1}{u^2}+c$

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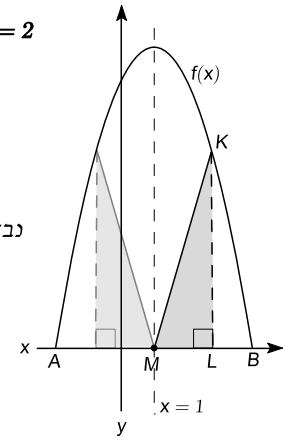
8. א.

$(I)\ \ f(-t)=-t^2-2t+c=0\ ,\ \ (II)\ \ f(2t)=-4t^2+4t+c=0$

$(I)-(II)\quad\Rightarrow\quad 3t^2-6t=0\quad/:3t_{(\neq 0)}\quad\Rightarrow\quad t-2=0\quad\Rightarrow\quad \boldsymbol{t=2}$

$(I)\quad -4-4+c=0\quad\Rightarrow\quad \boldsymbol{c=8}$

ב.



$\boldsymbol{f(x)=-x^2+2x+8}\quad\Rightarrow\quad x_M=-\frac{b}{2a}=-\frac{2}{-2}=1$

נבדוק עבור $x_K>x_M$. פתרון נוסף יהיה סימטרי לציר הסימטריה $x=1$.

$K(k,-k^2+2k+8)\quad\Rightarrow\quad ML=x_L-x_M=k-1$

$KL=y_K=-k^2+2k+8$

$S_{\triangle KLM}=\frac{(k-1)(-k^2+2k+8)}{2}=\frac{1}{2}(-k^3+3k^2+6k-8)=S(k)$

$S'(k)=\frac{1}{2}(-3k^2+6k+6)\overset{?}{=}0\quad/\cdot(-\frac{2}{3})\quad\Rightarrow\quad k^2-2k-2=0$

$k_{1,2}=\frac{2\pm\sqrt{12}}{2}=\frac{2\pm 2\sqrt{3}}{2}=1\pm\sqrt{3}\ ,\ x_K>x_M=1\quad\Rightarrow\quad x_K=1+\sqrt{3}$

$S''(k)=\frac{1}{2}(-6k+6)\quad\Rightarrow\quad S''(1+\sqrt{3})<0\quad\Rightarrow\quad max\ \ (\surd)$

הפתרון הנוסף, הסימטרי הוא $x=1-\sqrt{3}$ ולכן: $\boldsymbol{x_K=1\pm\sqrt{3}}$